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BULLETIN OF THE

AMERICAN MATHEMATICAL SOCIETY

THE SIXTH SUMMER MEETING OF THE AMERICAN MATHEMATICAL SOCIETY.

THE Sixth Summer Meeting of the AMERICAN MATHEMATICAL SOCIETY was held at the Ohio State University, Columbus, Ohio, on Friday and Saturday, August 25th and 26th, 1899. As has been the custom for several years, the meeting was held in conjunction with the American Association for the Advancement of Science. Aside from the material and incidental advantages, it has been found that the freedom of intercourse between the members of the Society and of Sections A and B of the Association, afforded by simultaneous meetings, serves to increase the interest in both gatherings, while the two-fold attachment of many members tends to enlarge the attendance. During the recent meeting of the American Association the initial steps were taken looking toward the still closer affiliation of the two societies for the purposes of the summer meeting, and the Committee of Section A was authorized to arrange for a joint meeting hereafter, if this should be deemed advisable.

The number of persons present during the meeting was over fifty, and included the following thirty-two members of the Society:

Professor W. W. Beman, Professor Henry Benner, Professor R. D. Bohannan, Professor C. H. Chandler, Professor L. E. Dickson, Professor C. L. Doolittle, Professor L. W. Dowling, Professor H. T. Eddy, Mr. H. B. Evans, Professor B. F. Finkel, Professor G. B. Halsted, Professor Thomas F. Holgate, Professor E. W. Hyde, Mr. H. G. Keppel, Professor T. E. McKinney, Professor A. MacFarlane, Professor W. H. Maltbie, Professor J. L. Markley, Professor Mansfield Merriman, Dr. G. A. Miller, Professor E. H. Moore, Professor A. B. Nelson, Professor H. B. Newson, Professor Alexander Pell, Professor G. T. Sellew, Professor J. B. Shaw, Dr. H. F. Stecker, Professor Ormond Stone, Dr. W. M. Strong, Professor A. L. P. Wernicke, Professor H. S. White, Professor R. S. Woodward.

The first session opened at ten o'clock on Friday morning, and was presided over by Professor R. S. Woodward, President of the Society. On taking the chair the presiding officer congratulated the Society on the manifest interest in mathematical study and investigation in this country as indicated by the long list of papers to be presented, and contrasted the present rapid growth of mathematical knowledge with the conditions twenty or twenty-five years ago.

By holding extended sessions the full programme was completed by Saturday noon. On Friday evening nearly all the members in attendance dined together at the Chittenden Hotel. This informal dinner has come to be a very interesting feature of the summer meeting and affords an opportunity for friendly interchange of thought upon mathematical topics that do not come up in the discussions of the regular programme.

At the Saturday morning session the Council announced the election of the following ten persons to membership in the Society: Professor Edgar Buckingham, Bryn Mawr College; Dr. Warren G. Bullard, University of Vermont; Mr. Walter B. Ford, Albany, N. Y.; Miss Fanny C. Gates, Woman's College, Baltimore; Miss Carrie Hammerslough, Columbia University; Professor Thomas F. Nichols, Hamilton College; Mr. Herbert W. Richmond, King's College, Cambridge, England; Professor George T. Sellew, Knox College, Galesburg, Illinois; Professor E. B. Skinner, University of Wisconsin; Dr. George P. Starkweather, Yale University. Nine applications for membership were received.

The following papers were read:

(1) Dr. A. S. CHESSIN: "Note on relative motion."

(2) Mr. F. R. MOULTON: "On surfaces of zero relative velocity and a certain class of special solutions in the problem of four bodies."

(3) Professor J. E. DAVIES: "On the use of generalized differentiation in the solution of physical problems."

(4) Professor ARNOLD EMCH: "A new class of link works."

(5) Mr. J. V. COLLINS: "A relation between point and vector analysis."

(6) Professor FLORIAN CAJORI: "John Speidell's 'New Logarithmes.'"

(7) Mr. H. W. RICHMOND: "On analogues of the property of the orthocenter."

(8) Professor H. S. WHITE: "Irrational covariant conics of a plane cubic."

(9) Professor FRANK MORLEY: "On the generalization of Desargues's theorem."

(10) Professor E. H. MOORE: "On certain crinkly curves."

(11) Dr. W. M. STRONG: "Note on non-quaternion number systems."

(12) Professor H. B. NEWSON: "On mixed groups."

(13) Dr. EDWARD KASNER: "The invariant theory of the inversion group."

(14) Miss E. N. MARTIN: "Note on the imprimitive substitution groups of degree fifteen, and on the primitive substitution groups of degree eighteen."

(15) Professor L. E. DICKSON: "A new definition of the general abelian linear group."

(16) Professor L. E. DICKSON: "Definition of various linear groups as groups of isomorphisms."

(17) Mr. W. B. FITE: "On the groups of cogredient isomorphisms that are abelian."

(18) Dr. G. A. MILLER: "On the groups that are direct products of two subgroups."

(19) Dr. G. H. LING: "A proof that there are no simple groups of order 1440, 1512, 1680, or 1800."

(20) Dr. H. F. STECKER: "On a relation between the totality of the elliptic functions and a line complex."

(21) Dr. VIRGIL SNYDER: "Geometric construction of the elliptic integral of the second kind, and of the Weierstrass sigma-function."

(22) Professor ALEXANDER PELL: "Some applications of elliptic functions."

(23) Dr. L. T. MORE: "On Fresnel's wave surface."

Mr. Moulton's paper was offered to the Society through Professor Moore, Mr. Collins's through Professor Benner, Dr. Kasner's through Professor Fiske, Miss Martin's through Professor Harkness, and Mr. Fite's through Dr. Miller.

On account of the absence of the authors, Dr. Chessin's paper was read by Professor Woodward, Mr. Moulton's and Professor Morley's by Professor Moore, Professor Davies's by Professor Hyde, Professor Emch's by Professor Newson, Professor Cajori's by the Secretary, Miss Martin's, Dr. Ling's and Dr. Snyder's by Dr. Miller, and Dr. More's by Professor Brace of the University of Nebraska. The papers by Mr. Richmond, Dr. Kasner, and Professor Dickson (No. 15) were read by title. Abstracts of the papers are given below.

Considering space motion in the theory of relative motion as a perturbation of the motion which the given system

would have if space motion did not exist, we arrive at the notion of a "perturbative function of relative motion." This function is the sum of three homogenous functions of p, q, r (components of the angular velocity ω of space rotation) of the respective degrees 0, 1, and 2. The purpose of Dr. Chessin's paper is to give the mechanical significance of the third of these three functions. The author shows that it is always possible to consider ω as the diagonal of a parallelogram constructed on two angular velocities ω_1 and ω_2, the direction of ω_1 being one about which relative rotation is *possible*, whereas such a rotation is *impossible* about the direction of ω_2. In this connection it was remarked that it is sometimes useful to express the kinetic energy of rotation of a body referred not to its *principal* axes of inertia, but to a system of *conjugate* axes. If then G denote the kinetic energy of space rotation,

$$2G = A'\omega_x^2 + B'\omega_y^2 + C'\omega_z^2,$$

where ω_x, ω_y, ω_z are the components of ω along and A', B', C' the moments of inertia about the conjugate (generally oblique) axes XYZ of the ellipsoid of inertia. Now, if we take for X the direction of ω_1, then

$$G = G_1 + G_2,$$
$$G_1 = \tfrac{1}{2} A'\omega_1^2, \qquad G_2 = \tfrac{1}{2} I\omega_2^2,$$

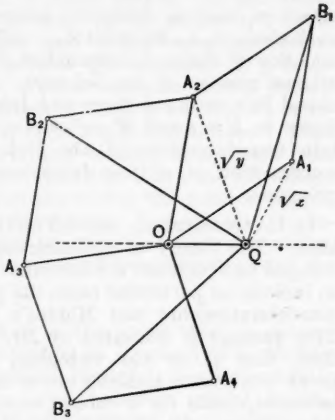
I denoting the moment of inertia about an axis in the plane conjugate with the direction of ω_1. Likewise, if we take for X the direction of ω_2, then

$$G_2 = \tfrac{1}{2} A'\omega_2^2, \qquad G_1 = \tfrac{1}{2} I\omega_1^2.$$

G_2 is exactly the third of the three terms of which ω is composed. Hence its mechanical significance.

Mr. Moulton's paper discussed the motions of four bodies, three of finite masses and one infinitesimal. The finite bodies were supposed to move according to one or other of the Lagrangian solutions, in circles around their common center of gravity. It was shown that the surfaces of zero relative motion separate those portions of space in which the infinitesimal body may move from those in which it cannot move. There are just twenty-eight solutions for which the system remains of constant form and size, and revolves in an arbitrary period.

The purpose of Professor Davies's paper was to call attention to some of the earlier investigations on the subject of generalized differentiation, and in particular to an apparently little known paper by Buchwaldt, a captain in the Danish army, who critically reviewed the work of Liouville, previously published in the *Journal de l'École Polytechnique*, for 1832, *et seq.* In view of some of the recent papers of Oliver Heaviside, wherein the usefulness of generalized differentiation in problems in electricity is pointed out, it was shown that attention had been called, long ago, by Liouville to the great value of this process in the solution of physical problems generally and that it is likewise valuable in analysis for certain inverse problems.



Professor Emch's paper, which is intended for publication in the *Annals of Mathematics*, is in abstract as follows:— The element of the linkwork is the *cell* $OA_1B_1A_2$-O-QB_1 in which O, Q are fixed points, and $OA_1 = A_1B_1 = B_1A_2 = A_2O$. Putting $QA_1 = \sqrt{x}$, $QA_2 = \sqrt{y}$, $QB_1 = R$, $OA_1 = r$, $OQ = c$, and $(R + r)^2 = \alpha$, $(r + c)^2 = \beta$, $(r - c)^2 = \gamma$, $(R - r)^2 = \delta$, the relation exists

$$\int^{y} \frac{dy}{\sqrt{(y-\alpha)(y-\beta)(y-\gamma)(y-\delta)}} - \int^{x} \frac{dx}{\sqrt{(x-\alpha)(x-\beta)(x-\gamma)(x-\delta)}} = a$$

where a is a constant. Let v and u designate the values of these integrals, so that $v - u = a$. The variables x and y may be expressed by elliptic functions

$$x = \lambda(u), \quad y = \lambda(v).$$

A second cell may be added having the same fixed points O and Q and the link OA_2 in common with the first cell. In

the same manner any number of cells may be added, thus forming a link-work of one degree of freedom. If the link-work consists of n cells, $\overline{OA_{n+1}}$ will be the last link and the argument belonging to $\overline{QA_{n+1}}$ will be $u + n \cdot a$. Supposing w_1 and w_2 as the periods of $\lambda(u)$, the quantities R, r, and c may be chosen in such a manner that

$$a = \frac{m_1 w_1 + m_2 w_2}{n}$$

where m_1 and m_2 designate integers. *In this case the link-work closes, i. e., the point A_{n+1} will coincide with A_1, and the condition of closing is independent of the value of u, or of the original position of the link-work.* In the special cases of closed link-works of three and four cells the condition reduces to $R = r$ and $R^2 - r^2 = r^2 - c^2$, respectively. A certain transformation of the link-work leads to Jacobi's construction of elliptic functions and Poncelet's poristic polygons.

In his Quaternions (second edition, page 22) Tait asserts that "The theory of 'geometrical nets' is very fully developed in Hamilton's Elements, and the method is shown to include as particular cases the processes of Grassmann's Ausdehnungslehre and Möbius's Barycentrische Calcul." This passage is criticized by Mr. Collins on two grounds: first, that it is too sweeping; and secondly, that if point and vector analysis are so closely connected, this relationship calls for a careful examination. The question arises, then, how can the two be brought together in a harmonious system. In his Elements, instead of studying point-vector analysis in its simplest form, Hamilton makes out of the simple expressions of point analysis very complex ones, anharmonic ratios in fact. In this way he limits the application of this analysis to the so-called modern geometry.

By Hamilton's ρ and Grassmann's p is or should be meant a complex concept consisting of a point p held in position by a vector from an origin ρ. Here p we will say denotes an infinitesimal portion of a line and $p\rho$ denotes p at the end of ρ whose initial point is at the origin. Thus $p\rho$ standing now for both Hamilton's ρ and Grassmann's p is really complex, just as ε ($= T\varepsilon U\varepsilon$ or $S\varepsilon + V\varepsilon$) is complex in quaternions, or as $p_1\varepsilon_1\varepsilon_2$ is complex in the Ausdehnungslehre. Using this notation the fundamental equations of point analysis become

$$(1) \qquad m_1 p\rho_1 + m_2 p\rho_2 = (m_1 + m_2)\, p\left(\frac{m_1\rho_1 + m_2\rho_2}{m_1 + m_2}\right),$$

$$(2) \qquad \Sigma mp\rho = \Sigma m \cdot p\left(\frac{\Sigma m\rho}{\Sigma m}\right),$$

$$(3) \qquad p\rho_2 - p\rho_1 = 0\, p\left(\frac{\rho_2 - \rho_1}{0}\right).$$

Hyde in his Directional Calculus writes these equations in duplicate, *i. e.*, first in the simple point analysis form, and then in the vector analysis form, the vectors being indicated as the difference of units points. If the above notation and equations be used in the solution of problems, the point equations and vector equations correspond and agree at each step of the solution. If p be divided out of any equation, a vector equation is left, while if the p's be given the subscripts of the ρ's, and the ρ's be removed, we have the regular point analysis equations. Thus this notation throws light on any solution or equation from two directions. The vector equations have the advantage that the origin can be taken anywhere; the point equations that they are always symmetrical. Because the fundamental concept dealt with in point analysis is complex, the notation used to present it should be complex until the subject is well grasped.

Professor Cajori remarked in his paper that no general history of mathematics published in England or in Europe makes mention of John Speidell, though it is quite generally known to English readers through certain special articles, for example, J. W. L. Glaisher's report on "Tables" in the British Association Report for 1873, that Speidell is the author of the earliest table of natural logarithms. The book appeared in 1619, only five years after Napier's famous work. A fac-simile of the title-page of the edition of 1622 was exhibited to the Society.

Mr. Richmond's memoir deals first with the systems of $2n$ points in a space of n dimensions S_n, such that the S_{n-1} determined by any n points is perpendicular to that determined by the remaining n points. Here for 'perpendicular' we may substitute 'conjugate as to a fixed quadric,' the absolute. More generally in S_n let there be p points $u_1 = 0, \cdots, u_p = 0$ and let the absolute be

$$\sum_1^p k_r u_r^2 = 0;$$

then any two spaces S_{n-1} which contain all the p points are necessarily perpendicular. For a proper quadric the values

of p range from $n+1$ to $2n$; for the euclidean absolute (passing over the case of points all at infinity) p ranges from $n+2$ to $2n$. Such a system of p points is called an orthogenic system. The system of $n+1$ orthogenic points is the self-conjugate pyramid; accordingly the system of $n+2$ points is selected for special consideration. This is the 'polviereck' in the case of the conic ($n=2$). and the 'polfünfeck' in that of the quadric in 3 dimensions. In the euclidean case, it is pointed out, *inter alia*, that the centroids of all pairs formed from the $n+2$ points lie on a sphere; and if further n be even, it is remarked that $n+1$ points determine uniquely $\frac{1}{2}n$ others, the whole forming an orthogenic system.

Professor White's paper embodied the results of some recent investigations upon a plane cubic curve and the systems of conics connected with it, and will be published in the *Transactions of the American Mathematical Society.* Two doubly infinite systems of conics are known (Hilbert in *Liouville*, 1888) which are related to the general plane cubic by an invariant property, namely, their equations are reproduced (to a factor) by an operation which transforms the cubic into its Hessian. Professor White finds each of these systems of conics to be the system of conic polars to a new cubic curve, and (1) these two cubics are coördinate with the fundamental cubic by virtue of a *cyclic* relation—each gives rise to the other two in the same way as the first did to the second and third; (2) all three lie in the syzygetic sheaf containing the first and its Hessian; (3) finally, the set of three are characterized by having their Cayleyan (curve of third class) in common. Two class-cubics defined by the self-reproductive property with respect to the transverse of this same operation prove to be coördinate to the conjugate class-cubic of the fundamental, and to be themselves conjugate to the two above described in the sense of Aronhold (*Crelle*, 55). They have the Cayleyan for their common Hessian. From these results, using the equation of the conjugate class-cubic, the covariant equations are written for conic polars of four cubics related to the Hessian just as the first four are to the Cayleyan.

Professor Morley communicated a proof of the theorem that all full spreads of order n in a linear space S_m which pass through

$$\frac{(m+n-1)!}{m!\,(n-1)!}+1$$

given points are apolar with a determinate full spread of class n in an intersecting space S_{m-1}. This paper is intended for publication in the *Transactions*.

Professor Moore's paper treats (1) of the area filling curves $x = \varphi(t)$, $y = \psi(t)$ of Peano and Hilbert, and in particular (2) of the curve $y = \psi(t)$, connected with the Peano curve, which is continuous and has at no point a progressive or a regressive non-vertical tangent, and at no point a progressive and a regressive vertical tangent. This curve from the simplicity of its geometric definition and from the intuitive clearness of its properties appears to be fit to replace the classical Weierstrass curve as the standard example of continuous curves having no tangents. The paper will be published in the *Transactions*.

Dr. Strong announced the following theorem: Any degenerate non-quaternion number system whose multiplication table contains m independent products can be reduced to a form such that the independent products will be $l_1, l_2, l_3, \cdots, l_m$, and will occur in the order here given. No independent product will be the product of units which themselves enter products. By 'degenerate system' is meant a system from which the modulus has been deleted; the term 'independent product' denotes a product of two units which is linearly independent of the products preceding it; and the 'order' of products in the multiplication table of non-quaternion number systems is defined thus: $l_i \cdot l_{i+a}$ and $l_{i+a} \cdot l_i$ precede $l_{i+b} l_{i+c}$ where $b, c \neq 0$; $l_i \cdot l_{i+a}$ and $l_{i+a} \cdot l_i$ precede $l_i \cdot l_{i+b}$ and $l_{i+b} \cdot l_i$ if $a < b$; $l_{i+a} l_i$ precedes $l_i l_{i+a}$ where $a \neq 0$.

The object of Professor Newson's paper was to exhibit a method for treating mixed groups of projective transformations (or collineations). The method was applied to one mixed group as an example. Mixed groups contain two distinct sets of transformations, one set forms a continuous group and the other set forms a multitude of discontinuous groups. All projective transformations of the plane which leave a pair of points A and B separately invariant form a continuous group $G(AB)$; add to these all transformations which interchange A and B, and we have the mixed group $mG(AB)$. All real projective transformations which interchange A and B were determined synthetically; they fall into four distinct classes. Those of the first class are involutoric, *i. e.*, of period two; those of the other three classes are non-periodic and are designated as elliptic, para-

bolic, and hyperbolic, respectively. The paper will be published in the *Kansas University Quarterly.*

To every group of transformations corresponds a geometry, or theory of invariants (Klein, Lie). The group considered by Dr. Kasner is the six-parameter group generated by the inversions of the plane; the corresponding inversion geometry is formally identical with projective geometry on a quadric surface. Two principal methods are developed for studying the invariant properties of geometric configuration. 1° Quaternary.—A curve whose equation in tetracyclic coördinates x_i connected by the quadratic identity $Q = 0$ is $f_n = 0$ is also represented by $f_n + MQ = 0$, where M is any form of order $n - 2$; but of the entire system of forms $f_n + MQ$, there is one and only one (F) which is apolar to Q. The final result may be stated: the (quaternary) inversion concomitants of the curve are the projective concomitants of F, Q. 2° Double binary.—A point being defined by the parameters $\lambda_1 : \lambda_2$, $\mu_1 : \mu_2$ of the pair of minimal lines passing through it, the equation of the curve to be studied takes the form $\varphi\ (\lambda_1^m \lambda_2,\ \mu_1^n \mu_2) = 0$; the (binary) inversion concomitants of the curve are the concomitants of the double binary form, φ, with respect to independent linear transformations of λ and μ. The relations between 1° and 2° were considered in detail. It was shown for example that the binary and quarternary polar theories coincide; and a series of methods was given of which 2° is the first member. Application was made to circles and cyclic curves.

In Miss Martin's paper it was shown that there are 68 imprimitive groups of degree fifteen. Of these 28 have their elements divisible into three systems of imprimitivity, 27 into five systems, while the remaining 13 have their elements divisible in both ways. In addition to the symmetric and to the alternating groups of degree eighteen only two other primitive groups have been found. One is a triply transitive group of order 18. 17. 16; the other is a doubly transitive simple group of order 16. 17. 8 contained self-conjugately in the former.

Professor Dickson's papers are intended for publication in the *Transactions.* An abstract giving some of the results of both papers was published in the BULLETIN, vol. 5, p. 331. Mr. Dickson also presented a preliminary communication containing an elementary proof of a theorem by Professor Moore on the abstract form of the symmetric group on k letters.

Mr. Fite's paper presented the following considerations: The group of cogredient isomorphisms of any group G cannot be the direct product of cyclical groups whose orders are such that one of them is not a divisor of the least common multiple of the others. If the group of cogredient isomorphisms is abelian, the order of every one of its operators is the order of a commutator of G. If G is of order p^m and contains an abelian subgroup of order p^{m-a}, its group of cogredient isomorphisms contains no operator of order greater than p^a, if it is abelian; if the largest abelian subgroup of G is of order p^{m-a}, where $a + \frac{1}{2}(m-1)$, G cannot have an abelian group of cogredient isomorphisms of order p^{m-1}. If G is of order p^m and contains an abelian subgroup of order p^{m-1}, it cannot have an abelian group of cogredient isomorphisms of order p^{m-a}, where $a < \frac{1}{2}(m-1)$.

Dr. Miller's paper, which will be offered for publication to the *Transactions*, is in abstract as follows:

If a group G contains two self-conjugate subgroups G_1, G_2 which have just K common operators, and if we form all the possible commutators $s^{-1}t^{-1}st$ by taking s from G_1 and t from G_2 (or vice versa) then these commutators generate a self-conjugate subgroup of G which is contained in K, and each operator of one of the two subgroups G_1, G_2 transforms every operator of the other into itself multiplied by some one of the given commutators. If G is generated by G_1 and G_2, and if $K = 1$, then G is said to be the direct product of G_1, G_2. Two theorems are proposed:

THEOREM I. If the order g of a group G is divisible by p^a but not by p^{a+1}, p being any prime number, and if all the conjugates of any operator (s) contained in a subgroup P of order p^a may be obtained by transforming s by means of operators of P, then G is the direct product of P and some other subgroup of G, and it contains a self-conjugate subgroup of each of the orders $g \div p^\beta$ ($\beta = 0, 1, 2, \ldots, a-1$).

THEOREM II. If the order of a group is mn (m being prime to n) and if it contains a subgroup of order m which has the property that each of its operators is transformed by some one of the operators of this subgroup in the same way as it is transformed by any given operator of the entire group, then the group is the direct product of two subgroups whose orders are m and n respectively.

Hölder, Cole, and Burnside have examined all the orders up to 1093 in regard to possible simple groups and proved

that there are no simple groups of any of these orders with exception of those already known whose orders are 60, 168, 360, 504, 660 and 1092. At the February meeting of this Society Dr. Ling presented a paper in which all the other orders that do not exceed 2000, except four, were examined. The orders 1440, 1512, 1680 and 1800 presented difficulties which could not then be overcome. With the assistance of Dr. Miller, Dr. Ling has now examined these orders and the result reached is that *there is no simple group whose order lies between the numbers* 1092 *and* 2001. These results will be incorporated in the paper previously presented to the Society and will be offered to the *American Journal of Mathematics* for publication.

The fundamental considerations in Dr. Stecker's paper were these: (1) the six line coördinates of a complex satisfy the identical relation $p_1p_2 + p_3p_4 + p_5p_6 = 0$, and conversely, any six quantities satisfying such an identical relation may be treated as line coördinates of a complex; (2) the biquadratic which enters into an elliptic integral has three irrational invariants each of which is the product of two invariant determinants. These three irrational invariants, A, B, C, satisfy the relation $A + B + C = 0$. Hence the above-mentioned six determinants may be treated as line coördinates of a complex. Several interesting results were pointed out.

Dr. Snyder's paper describes the models of the functions $\sigma(u)$ and $\zeta(u)$, recently prepared by the author under the direction of Professor Klein. These models, which are to be reproduced in plaster, are constructed with the same constants ($g_2 = 0$, $g_3 = 4$) employed by Professor Burkhardt in the case of $\wp(u)$ and $\wp'(u)$, thus completing the set to a certain extent.

Professor Pell's paper dealt with the curves on an ellipsoid, called by E. Cosserat the 'D' lines. The author expressed the coördinates of these lines for the ellipsoid as uniform functions of one parameter; by means of these expressions he traced the path of the curves, and deduced some interesting properties of these lines. The paper will be offered for publication to the *Transactions*.

Dr. More pointed out in his paper that it is generally assumed that unless a ray of light in a biaxial crystal is refracted along an optic axis, it will break up into two rays—the ordinary and the extraordinary—but the fact is that there are an infinite number of directions along which the ray is not

divided. Suppose the section of Fresnel's wave surface by a coördinate plane to consist of a circle and an ellipse and let O be the center of a wave disturbance in the surface of a crystal. Draw any radius vector OBC, cutting the circle at B and the ellipse at C; the intersection of tangents at B and C will be S, a point on the tangent plane at the point of incidence. Then SB and SC will be the wave fronts of an ordinary and an extraordinary ray. Since these two lines are tangents at the points of intersection of a single radius vector OBC. the latter is the path of both an ordinary and an extraordinary ray. Therefore from the indices of refraction we may compute an incident ray which will not be divided but will move along OBC, and since there are an infinite number of points S there are an infinite number of directions for which the ray is not divided. The locus of the point S is a curve of the eighth degree and may be expressed in a simple and symmetrical form. By a simple transformation of the equation uniaxial crystals are included in this treatment.

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Acting Secretary.

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REPORT ON THE RECENT PROGRESS IN THE THEORY OF LINEAR GROUPS.

BY PROFESSOR L. E. DICKSON.

(Read before Section A of the American Association for the Advancement of Science, Columbus, Ohio, August ?, 1899.)

THIS report is intended to supplement in certain directions the report* of Dr. Miller on group theory read at the last summer meeting of the American Association. In order to allow space for important details, I will consider neither continuous nor discontinuous linear groups of infinite order, but confine my report to finite linear groups. Of these I shall consider first the finite collineation groups and afterwards the linear congruence groups and the more general linear groups in Galois fields.

PART I.—FINITE GROUPS OF LINEAR SUBSTITUTIONS.

§1. *General Theorems on Finite Linear Groups.*

The following theorem was stated by Jordan in 1878 and proven in various ways by Lipschitz, Kronecker, Rost,

* BULLETIN, Feb., 1899, p. 227.

Maschke, Moore,* and recently by Burnside† in a very elementary manner :

A linear homogeneous substitution of finite period p,

$$x_i' = \sum_{j=1}^{n} a_{ij} x_j, \qquad (j = 1, 2, \cdots, n)$$

may be linearly transformed to the canonical form

$$y_i' = \varepsilon_i y_i, \qquad (i = 1, 2, \cdots, n)$$

where each ε_i is a p^{th} root of unity.

Concerning the nature of the irrationalities which may enter into the coefficients of the substitutions of a finite linear group, we have the theorem due to Professor Maschke :—‡

If a linear substitution group of finite order contain at least one substitution, the roots of whose characteristic equation are all distinct, the group can be so transformed that all the coefficients of the substitutions of the transformed group are cyclotomic, i. e., are rationally expressible by roots of unity.

The existence of a universal invariant for finite linear groups is expressed by the theorem :—

A finite group of n-ary linear homogeneous substitutions leaves absolutely invariant a positive Hermitian form. By suitable linear transformation of the group this invariant becomes

$$\sum_{i=1}^{n} x_i \overline{x_i}.$$

This theorem was stated by Loewy § and proven by Moore.‖ The first part of the theorem was established earlier by Picard ¶ for the cases $n = 2, 3$. The complete theorem was proven by Valentiner ** for $n = 2$ and 3.

H. Minkowski †† has shown that every finite group G of n-ary linear homogeneous substitutions with integral coefficients is isomorphic with the group of substitutions re-

* *Math. Annalen*, vol. 50, pp. 213–219. Full references are given in the paper by Professor Moore.

† *Messenger of Math.*, no. 331 (November, 1898).

‡ *Math. Annalen*, vol. 50, p. 492.

§ *Comptes rendus*, vol. 123 (1896), pp. 168–171.

‖ *Math. Annalen*, vol. 50, pp. 213–219.

¶ *Bull. Soc. Math de France*, vol. 15 (1887), pp. 152–156.

** "De endelige Transformations-Gruppers Theori," avec un resumé en français, *Danish Academy*, 1889.

†† *Crelle*, vol. 100 (1887), pp. 449–458.

sulting upon reducing its coefficients modulo 2, and derives the corollary that the order of G is a divisor of $2^n N$, where $N \equiv (2-1)\ (2^n-2) \cdots (2^n-2^{n-1})$ is the order of the group of all n-ary substitutions taken modulo 2.

The generators of the group of n-ary linear substitutions of determinant unity and with integral coefficients were given by Kronecker* as follows:

$$(1) \qquad x_1 = -x_k', \quad x_k = x_1', \quad x_i = x_i' \quad (i \gtrless k,\ i > 1),$$

$$(2) \qquad x_1 = x_1' + x_2', \qquad x_i = x_i' \qquad (i = 2, 3, \cdots, n).$$

In place of these n generators, Krazer† uses three generators. But Kronecker had already shown that two generators would suffice, while the simple form of the generators would thereby be lost. For the case $n = 3$, W. Burnside‡ proves that the generator (2) together with the substitution permuting x_1, x_2, x_3 in a cycle suffices to generate the group. In Kronecker's paper the generators of the abelian linear groups are also given.

§ 2. *Special Groups of Collineations.*

The problem of the determination of all finite groups of binary linear substitutions has been solved geometrically by Klein§ and algebraically by Gordan.‖ Essentially the groups thus found are those connected with the regular solids. The finite ternary groups were considered at great length by Jordan,¶ who, however, missed the group of 168 collineations in the plane, which was derived by Klein** by the consideration of the transformation of the seventh order of elliptic functions. He also missed the simple group of 360 collineations, set up by Valentiner†† and proven simply isomorphic to the alternating group on six letters by Wiman.‡‡ Certain configurations connected with this G_{360} had been given earlier by Gerbaldi.§§

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* *Monatsber. der Berl. Akad.* (1866), p. 597.

† *Annali di Matematica*, 2d series, vol. 12, pp. 283–300.

‡ *Messenger of Math.*, 2d series, vol. 24 (1894), pp. 109–112.

§ *Math. Annalen*, vol. 9 (1876), pp. 183–188.

‖ *Math. Annalen*, vol. 12 (1877), pp. 23–46.

¶ *Crelle*, vol. 84 (1878), pp. 89–215; *Atti della Reale Accademia di Napoli*, vol. 8 (1879).

** *Math. Annalen*, vol. 14, pp. 428–471 (Jordan's omission is indicated in the foot-note to p. 438); *Math. Annalen*, vol. 15, pp. 251–282.

†† "De endelige Transformations-Gruppers Theori," avec un résumé en français, 1889, *Danish Academy.*

‡‡ *Math. Annalen*, vol. 47.

§§ "Sul gruppo semplice di 360 collineazioni piane," *Math. Annalen*, vol. 50, p. 472.

In view of the evident independence of the work of Jordan and Valentiner and their difference of method, we may feel certain of the completeness of the determination of the finite ternary linear groups. Of these groups the ones which leave a straight line invariant as well as those permuting three points cyclically are regarded as trivial. Besides these there exist only the groups of orders 216, 168, 360, together with their subgroups of orders 72, 36, and 60. We will briefly characterize the former set of three groups.

The Hesse group of 216 collineations is defined by a pencil $\lambda F + \mu H$, where F is a ternary cubic and H its Hessian covariant. The G_{216} has subgroups of orders 36 and 72. It has been studied at length by Maschke.*

The simple group of 168 collineations leaves invariant

$$x^3y + y^3z + z^3x = 0.$$

As mentioned above, it was first met by Klein. It is studied in detail in Klein-Fricke's Modulfunctionen. The types of substitutions and of subgroups of the G_{168} are given by Gordan† (*Mathematische Annalen*, vol. 25, p. 462).

The simple group of 360 collineations has as its simplest invariant form the function

$$10x^3y^3 + 9z(x^5 + y^5) - 45x^2y^2z^2 - 135xyz^4 + 27z^6.$$

It has two systems, each of 15 conjugate octahedral subgroups, and two systems each of six conjugate icosahedral subgroups. The even permutations of the icosahedral groups of one such system form the alternating group on six letters (Wiman). The G_{360} has ten conjugate subgroups of order 36. It is not possible to set up a group of 360 ternary homogenous substitutions holoedrically isomorphic with the group of 360 collineations.‡ This result follows from the corresponding impossibility for the subgroup G_{36}, as shown by Jordan.

Quaternary linear homogenous groups of orders 120 and 720 have been given by Klein,§ one of order $2 \cdot 16 \cdot 6!$ was studied by Reichardt.¶ All ternary and quaternary linear groups simply isomorphic to the symmetric or alternating group of substitutions have been determined by Maschke.**

* *Math. Annalen*, vol. 33.

† "Ueber Gleichungen siebenten Grades mit einer Gruppe von 168 Substitutionen:" I. *Math. Annalen*, vol. 20, pp. 516–530; II. *Math. Annalen*, vol. 25, pp. 459–521.

‡ Wiman, *Math. Annalen*, vol. 47, p. 538.

§ Ikosaeder, p. 198; *Math. Annalen*, vol. 4.

¶ *Leipzig Ber.* (1885), pp. 419–426.

** *Math. Annalen*, vol. 51.

The problem emphasized by Klein of representing every group as a collineation group in the fewest possible number of variables has been solved for the symmetric and alternating groups by A. Wiman.* He shows that, for $n>7$, no collineation group exists in space of $<n-2$ dimensions, which is isomorphic with the symmetric or alternating group on n letters. Except for the case $n=9$, there is only one group in R_{n-2} isomorphic with the symmetric and one with the alternating group on n letters.

A quaternary linear homogeneous group of order 51840 arising in the theory of the transformation of hyperelliptic functions has been studied by Klein, Burkhardt, Maschke, Witting, and others. Jordan † had already determined the Galois group of the equation for the 27 lines on a general cubic surface and proved‡ it isomorphic with the abelian group for the trisection of hyperelliptic functions of four periods. The possibility of the solution by hyperelliptic functions of the equation for the 27 lines on a cubic surface having been thus established, Klein§ proceeded to sketch a method of reduction of the problem of the equation of the 27 straight lines to the problem of the trisection of the hyperelliptic function of 1st order. The actual calculations were effected by Burkhardt,|| who gives full references to the work of Witting¶ on a configuration in space connected with the G_{51840} and to that of Maschke** on the complete form system of the group.

From the obscurity of a passage on p. 214 of a note by G. G. Morrice,†† some readers have inferred that the simplicity of the group of 51840 linear substitutions had been established. The group probably meant was the group of order 4 and the word "simplicity" is not used in its technical group sense.

Various linear congruence groups will be found in Part II which are isomorphic with the present G_{51840}. The latter group is a subgroup of index 28 under the group of order 1,451,520, the group of the equation of the 28th de-

* *Math. Annalen*, vol. 52 (1899), pp. 243-270 ; *Gött. Nach.* 1897.

† Traité des substitutions, pp. 316-329 ; also Kühnen's Dissertation, Marburg, 1888.

‡ *Ibid*, pp. 365-369 ; a shorter proof has been given by the writer in *Comptes rendus*, vol. 128 (1899), pp. 873-5.

§ *Liouville's Journal* (4), vol. 4 (1888), pp. 169-176.

|| *Math. Annalen*, vol. 38, and vol. 41, p. 312.

¶ Gött. Diss., 1887 ; *Math. Annalen*, vol. 29.

** *Math. Annalen*, vol. 33, pp. 317-344.

†† *Proc. Lond. Math. Soc.*, vol. 22 (1892), pp. 213-217.

gree on which depends the theory of the double tangents to a curve of the fourth order.*

The determination of all surfaces of the third order which admit of collineations has been made by Karl Bobek.† The only surfaces found were those studied by Eckhardt.‡ Among these is Clebsch's diagonal surface§ with the group G_{120}; also surfaces with a G_{648} and a G_{108}.

In the paper of Biermann,‖ "Ueber die regelmässigen Punktgruppen in Räumen höherer Dimension und die zugehörigen linearen Substitutionen mehrerer Variabeln," Klein's method for ordinary space is extended to n dimensions and worked out for the case $n = 5$. Schlegel had already determined the regular bodies in R_5. Biermann reached linear substitutions in two complex variables previously studied by Jordan and Klein.

Two special classes of finite linear groups of orders n^3 and $n^4(n^2 - 1)$ are given by Jordan.¶

Part II.—Linear Groups in a Galois Field.

§3. *Theory of the Galois Field.*

In a great many investigations in group theory, including the classic ones of Galois, Betti, Mathieu, Jordan, as well as in numerous more recent articles, the theory of Galois's imaginaries has proved to be an instrument of great power. Galois first studied the properties of the polynomials built on a root of a congruence of degree n belonging to and irreducible modulo p. These p^n distinct polynomials combine according to the ordinary laws of algebra. Let for example j denote an (imaginary) root of the irreducible congruence

$$x^3 + x + 1 \equiv 0 \pmod 2.$$

Then all rational functions of j can be reduced to one of the eight forms

$$0, \quad 1, \quad j, \quad j+1, \quad j^2, \quad j^2+1, \quad j^2+j, \quad j^2+j+1.$$

The sum, difference, product, or quotient (except by zero) of any two of these polynomials reduced to an unique polynomial belonging to the set.

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* Geiser, *Math. Annalen*, vol. 1, pp. 129–138.
Jordan, Traité des substitutions, p. 330; *Comptes rendus*, vol. 67 (1868).
Weber, *Math. Annalen*, vol. 23, pp. 489–503.
† *Monatshefte für Math. und Physik.* (Wien), vol. 10 (1899), pp. 102–122.
‡ *Math. Annalen*, vol. 10, p. 227.
§ *Math. Annalen*, vol. 4, p. 336.
‖ *Wien Ber.*, vol. 95, pp. 523–548.
¶ *Bull. S. M F.*, vol. 5, pp. 175–177.

Moore's abstract formulation of the theory is convenient for group investigations. In place of the above set of eight polynomials, Moore considers eight abstract *marks* constituting the Galois field of order 8, in notation the $GF[2^3]$.

Rather elaborate expositions of the theory have been given by Schönemann, Dedekind, Serret, and Moore. A very complete, but elementary, exposition of the abstract theory of Galois fields, with proof of their existence, will be published shortly by the writer. Owing to the lack of references in the papers on the subject, the following bibliography on Galois fields has been added:

GALOIS: "Sur la théorie des nombres," *Bulletin des sciences mathématiques*, de M. Férussac, 1830; reprinted in *Journal de mathématiques*, 1846, and in Oeuvres mathématiques d'Evariste Galois, Paris, 1897.

SCHÖNEMANN: "Grundzüge einer allgemeinen Theorie der höheren Congruenzen deren Modul eine reelle Primzahl ist," *Crelle*, vol. 31 (1846), pp. 269–325.

DEDEKIND: "Abriss einer Theorie der höheren Congruenzen in Bezug auf einen reellen Primzahl-Modulus." *Crelle*, vol. 54 (1857), pp. 1–26.

SERRET: "Sur les fonctions entières irréductibles suivant un module premier, dan le cas où le degré est une puissance du module," *Journal de Liouville*, 1873, p. 301 and p. 437; also in Cours d'algèbre supérieure, vol. 2, pp. 122–211.

JORDAN: "Théorie de Galois," Traité des substitutions, 1870, pp. 14–18.

PELLET: "Sur les fonctions irréductibles suivant un module premier et une fonction modulaire," *Comptes rendus*, vol. 70 (1870), pp. 328–330, and vol. 93 (1881), p. 1065, also vol. 90 (1880), p. 1339.

PELLET: "Sur les fonctions réduites suivant un module premier," *Bulletin Soc. Math. de France*, vol. 17 (1889), pp. 156–167.

PELLET: "Sur la décomposition d'une fonction entière en facteurs irréductibles suivant un module premier p," *Comptes rendus*, vol. 86 (1878), p. 1071.

MOORE: "A doubly-infinite system of simple groups," Congress Mathematical Papers, 1893.

BOREL et DRACH: Théorie des nombres et algèbre supérieure, 1895.

DICKSON: "Higher irreducible congruences," BULLETIN, July, 1897, pp. 381–389.

§4. *The General Linear Homogeneous Group.*

Consider the p^{nm} letters $l_{\xi_1, \xi_2, \cdots, \xi_m}$ characterized by m indices, each being an arbitrary one of the p^n marks of $GF[p^n]$. A *linear homogeneous substitution* (A) on these letters replaces $l_{\xi_1, \cdots, \xi_m}$ by $l_{\xi_1', \cdots, \xi_m'}$ where

$$\text{(A)} \qquad \xi_i' = \sum_{j=1}^{m} a_{ij}\xi_j \qquad (i = 1, \cdots, m),$$

the a_{ij} being certain marks of the $GF[p^n]$, such that the determinant $|a_{ij}| \neq 0$.

Since the compound of two linear homogeneous substitutions is again a linear homogeneous substitution, we say that the totality of substitutions (A) form a group, the *general linear homogeneous group of degree* p^{nm}. In the linear form, this group is due to Jordan (Traité des substitutions, § 119, § 169), special forms being due to Galois. In the writer's dissertation* it was proven to be identical with a group of substitutions in one variable investigated by E. Betti in 1852–1855 and in more detail by E. Mathieu in 1861. Its structure (factors of composition) was determined by Jordan for the case $n = 1$, and for general n by the writer in his dissertation and somewhat later by Burnside.†

§5. *The Linear Fractional Group.*

It is obtained analytically by taking the linear homogeneous group fractionally, so that its substitutions may be exhibited in the form

$$\xi_1' : \xi_2' : \cdots : \xi_m' = \sum_{j=1}^{m} a_{1j}\xi_j : \sum_{j=1}^{m} a_{2j}\xi_j : \cdots : \sum_{j=1}^{m} a_{mj}\xi_j.$$

It is a concrete form of the quotient group of the general linear homogenous group by the invariant subgroup formed of the substitutions.

$$\xi_i' = \mu\xi_i, \qquad (i = 1, \cdots, m).$$

For the case $m = 2$, we have the ordinary linear fractional group in one variable. It was investigated by Galois, Mathieu,‡ etc.; its structure was determined for the case $n = 1$ by Gierster‖ and for general n by Professor Moore§

* *Annals of Math.*, 1897, pp. 65–120, pp. 161–183.

† The theory of groups, pp. 340–341.

‡ *Liouville's Journal* (2) vol. 5, pp. 9–42; vol. 6, pp. 241–323.

‖ *Math. Annalen*, vol. 18 (1881), pp. 319–365.

§ The Congress Math. Papers of 1893; abstract in the BULLETIN, Dec. 1893.

and later, but imperfectly, by Professor Burnside.* The subgroups of the linear fractional group in the $GF[p^n]$ were determined by Gierster for $n = 1$ (*Mathematische Annalen*, vol. 18 (1881), pp. 319–365) and for general n by Moore (Abstract in the BULLETIN, Oct., 1898). It was shown that the subgroups of the modular group are either solvable groups or certain modular groups or finally groups of double the order of certain modular groups. The decomposition of the subgroups of the modular groups thus leads only to cyclic groups and modular groups.

The linear homogeneous group was proven by Moore (BULLETIN, Nov., 1895) to be the group of holoedric transformation into itself of the regular abelian group of degree p^n. In the same paper were determined certain abstract defining invariants of the linear homogeneous and the linear fractional groups.

§ 6. *The Abelian Linear Group.*

Consider two sets of $2m$ indices

$$\xi_i, \quad \eta_i; \quad \xi_a, \quad \eta_a \qquad (i = 1, 2, \cdots, m).$$

Of the substitutions of the general linear homogeneous group on $2m$ indices in the $GF[p^n]$, those which, if operating simultaneously on the two sets of indices, multiply the function

$$\zeta \equiv \sum_{i=1}^{m} \begin{vmatrix} \xi_i & \xi_a \\ \eta_i & \eta_a \end{vmatrix}$$

by a constant factor, form a group, the general abelian group. We exclude the case $m = 1$, when the group is merely the general binary linear homogeneous group treated in § 4.

Its structure was determined by Jordan † for the case $n = 1$ and by the writer ‡ for general n. Indeed, its factors of composition are the prime factors of $p^n - 1$ together with the factors of composition of the special abelian group of those substitutions leaving ζ absolutely invariant. For $p = 2$, the latter group is simple except when $m = 2$, $n = 1$, when it is isomorphic with the symmetric group on six letters. For $p > 2$, it has a maximal self-conjugate subgroup of order 2, so that the quotient group is simple.

§ 7. An immediate generalization of the abelian group consists in the group of substitutions on mq variables

* *Proc. Lond. Math. Soc.*, vol. 25 (Feb., 1894). pp. 113–139,.

† Traité des substitutions, pp. 171–179.

‡ *Quar. Jour. of Math.*, July, 1897, pp. 169–178, and April, 1899, pp. 383–4.

$$x_{ij}' = \sum_{k=1}^{m} (a_{k1}^{ij} x_{k1} + \cdots + a_{kq}^{ij} x_{kq}) \qquad \begin{pmatrix} i=1,\cdots,m \\ j=1,\cdots,q \end{pmatrix}$$

which, if operating cogrediently upon q independent sets of mq variables, the j th set of which is exhibited thus

$$x_{i1}^{(j)}, \quad x_{i2}^{(j)}, \quad \cdots, \quad x_{iq}^{(j)} \qquad (i = 1 \cdots m)$$

will leave invariant up to a constant factor the function

$$\varphi = \sum_{i=1}^{m} \begin{vmatrix} x_{i1}^{(1)} & x_{i2}^{(1)} \cdots & x_{iq}^{(1)} \\ x_{i1}^{(2)} & x_{i2}^{(2)} \cdots & x_{iq}^{(2)} \\ \cdots & \cdots & \cdots \\ x_{i1}^{(q)} & x_{i2}^{(q)} & x_{iq}^{(q)} \end{vmatrix}.$$

This group was indicated by Kronecker.† The continuous group of linear substitutions leaving φ invariant was investigated by the writer in the BULLETIN for May, 1897. The structure of the discontinuous group of substitutions in the $GF[p^n]$ which leave φ absolutely invariant has been determined by the writer.* For $q > 2$, a substitution leaving φ invariant must leave each determinant in φ separately invariant or at most permute these determinants.

§8. *The First and Second Hypoabelian Groups.*

These groups are certain subgroups of the abelian linear group in the $GF[2^n]$. For $n = 1$, they were defined and investigated at length by Jordan making use of his theory of "exposants d'échange."

By a suitable generalization of Jordan (Traité, p. 440, § 578), we can give the following elementary definition of the generalized hypoabelian groups. The first and second hypoabelian groups in the $GF[2^n]$ are the groups of linear homogeneous substitutions

$$\begin{aligned} \xi_i' &= \sum_{j=1}^{m} (\alpha_{ij}\xi_j + \gamma_{ij}\eta_j) \\ \eta_i' &= \sum_{j=1}^{m} (\beta_{ij}\xi_j + \delta_{ij}\eta_j) \end{aligned} \qquad (i = 1, \cdots, m)$$

which leave invariant the respective functions

$$\sum_{i=1}^{m} \xi_i\eta_i, \quad \sum_{i=1}^{m} \xi_i\eta_i + \lambda(\xi_1^2 + \eta_1^2),$$

where λ is a mark such that $\xi_1\eta_1 + \lambda(\xi_1^2 + \eta_1^2)$ is an irreducible quadratic form in the field. The orders of the groups are respectively

† Compare Jordan, Traité des substitutions, p. 219.

* "A class of linear groups including the abelian group," *Quar. Jour. of Math.*, July, 1899.

$$2(2^{2m} \mp 1)[(2^{2n(m-1)} - 1)2^{2n(m-1)}]$$
$$[(2^{2n(m-2)} - 1)2^{2n(m-2)}] \cdots [(2^{2n} - 1)2^{2n}].$$

Each group has a subgroup of index two, defined by the invariant equation

$$\sum_{i,j}^{1 \cdots m} \alpha_{ij}\delta_{ij} + \lambda^2(\alpha_{11}^2 + \beta_{11}^2 + \gamma_{11}^2 + \delta_{11}^2) = m,$$

the terms in λ^2 not occurring for the first hypoabelian group. The subgroup is simple in the case of the first hypoabelian group if $m > 2$, and for the second hypoabelian group if $m > 1$. For $m = 2$,* the former has two factors of composition each equal to $(2^{2n} - 1)2^n$.

A definition of the abelian and the two hypoabelian groups as groups of isomorphisms of certain elementary groups has been given by the writer.† Consider, for example, the abstract group F generated by the operators $\theta, A_1, \cdots, A_m, B_1, \cdots, B_m$, all of period p and mutually commutative except the pairs A_i, B_i for which

$$A_iB_i = \theta B_iA_i \qquad (i = 1, 2, \cdots, m).$$

The group of isomorphisms of F is isomorphic with the abelian linear group for $p > 2$ and with the first hypoabelian group for $p = 2$.

§9. *The Orthogonal Group.*

This group is formed of the linear homogeneous substitutions on m indices $\xi_1, \ldots, \xi_m$ in the $GF[p^n], p > 2$, which leave invariant the function $\sum_{i=1}^{m} \xi_i^2$. Its order and generators were determined by Jordan‡ for the case $n = 1$, certain exceptional cases being overlooked, and by the writer§ for general n. It is generated by the following substitutions [only those indices which are altered are written]:

$$C_i: \quad \xi_i' = -\xi_i,$$

$$O_{ij}^{\alpha\beta}: \quad \begin{cases} \xi_i' = \alpha\xi_i + \beta\xi_j, \\ \xi_j' = -\beta\xi_i + \alpha\xi_j, \end{cases} \qquad (\alpha^2 + \beta^2 = 1),$$

with two exceptional cases, when additional generators are necessary, viz.,

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* *Amer. Jour. of Math.*, July, 1899, p. 249.
† Abstract in the BULLETIN, April, 1899; complete in the *Transactions*.
‡ Traité des substitutions, pp. 155-177.
§ BULLETIN, Feb., 1898.

$$\text{for } p^n = 5,\ m \equiv 3, \qquad R: \left\{ \begin{matrix} \xi_1' = \xi_1 + \xi_2 + 2\xi_3 \\ \xi_2' = \xi_1 + 2\xi_2 + \xi_3 \\ \xi_3' = 2\xi_1 + \xi_2 + \xi_3 \end{matrix} \right\} \quad (R^2 = 1),$$

$$\text{for } p^n = 3,\ m \equiv 4, \qquad W: \left\{ \begin{matrix} \xi_1' = \xi_1 - \xi_2 - \xi_3 - \xi_4 \\ \xi_2' = \xi_1 - \xi_2 + \xi_3 + \xi_4 \\ \xi_3' = \xi_1 + \xi_2 - \xi_3 + \xi_4 \\ \xi_4' = \xi_1 + \xi_2 + \xi_3 - \xi_4 \end{matrix} \right. \quad (W^2 = 1).$$

Its structure has been fully determined by the writer in several distinct manners.* For m odd and > 1, the orthogonal group has the factors of composition 2, 2, and

$$\tfrac{1}{2}(p^{n(m-1)} - 1)\, p^{n(m-2)}\, (p^{n(m-3)} - 1)\, p^{n(m-4)} \cdots (p^{2n} - 1)\, p^n,$$

the case $p^n = 3$, $m = 3$ being an exception, the factors being 2, 2, 2, 2, 3.

For m even and > 4, it has the factors of composition 2, 2, 2, and

$$\tfrac{1}{4}[p^{n(m-1)} - \varepsilon^{\frac{m}{2}} p^{n(\frac{m}{2}-1)}]\ [(p^{n(m-2)} - 1)\, p^{n(m-3)})]$$
$$\cdots [(p^{2n} - 1)\, p^n],$$

ε being ± 1 according to the form $4l \pm 1$ of p^n.

For $m = 4$, it has the factors of composition 2, 2, $\frac{1}{2}(p^{2n} - 1)\, p^n$, $\frac{1}{2}(p^{2n} - 1)\, p^n$ and 2, except in the cases $p^n = 2$ and 3, when the composite numbers 6 and 12 respectively are to be replaced by their prime factors.

§10. *Other Linear Groups with a Quadratic Invariant.*†

Every m-ary linear group in the $GF[p^n]$, $p > 2$, defined by an invariant which is a quadratic form of determinant not zero in the field, can be transformed into the orthogonal group if m be odd; while for m even, it can be transformed either into the orthogonal group or else into the group leaving invariant the form

$$\sum_{i=1}^{m-1} \xi_i^2 + \nu \xi_m^2 \qquad (\nu = \text{not-square}).$$

The latter group has for $m > 2$, the factors of composition 2, 2, and

$$\tfrac{1}{2}[p^{n(m-1)} + \varepsilon^{\frac{m}{2}} p^{n(\frac{m}{2}-1)}]\ [(p^{n(m-2)} - 1)\, p^{n(m-3)})]$$

* *Proc. Calif. Acad. Sci.* (3), vol. 1, nos. 4-5; BULLETIN, May, 1898; *Amer. Jour. of Math.*, July, 1899; *Proc. Lond. Math. Soc.*, vol. 30, pp. 70-98.

† *Amer. Jour. of Math.*, July, 1899.

$$\cdots [(p^{2n}-1)\,p^n].$$

For $m=2$, it is simply isomorphic with the linear fractional group in the $GF[p^{2n}]$.

Every m-ary linear homogeneous group in the $GF[2^n]$ defined by a quadratic invariant, not expressible as a quadratic form in fewer than m variables, can be transformed linearly into a group which, for m even, is either the first or else the second hypoabelian group; while for m odd, $m \equiv 2M+1$, it is the group leaving invariant

$$\xi_0^2 + \sum_{i=1}^{M} \xi_i \eta_i.$$

The latter group is simply isomorphic with the abelian group on $2M$ indices in the $GF[2^n]$.

§ 11. *Linear Group defined by the Invariant* $\sum_{i=1}^{m} \xi_{i_1} \xi_{i_2} \cdots \xi_{i_q}$.

For $q=2$, this group was studied at length by the writer (*Proc. Lond. Math. Soc.*, vol. 30, pp. 70–98), and for $q>2$ (Ibid., pp. 200-208). For $q=2$, $p>2$, the group is isomorphic with the orthogonal group, except in the case m odd and -1 a not-square in the field, when it is isomorphic with the group of § 10. For $q>2$, the substitutions leave the products $\xi_{i_1} \xi_{i_2} \cdots \xi_{i_q}$ $(i=1, \cdots, m)$ separately invariant, or else merely permute them. Its structure is therefore evident.

§ 12. *The Hyperorthogonal Group.*

In a paper contributed to the *Mathematische Annalen*, the writer has determined the order, generators, and structure of the linear homogeneous m-ary group defined by the invariant

$$\varphi_r \equiv \sum_{i=1}^{m} \lambda_i \xi_i^r \qquad (r>2).$$

There exists no continuous group leaving invariant φ_r $(r>2)$, while the only collineations of $\varphi_r = 0$ into itself are those leaving the terms $\lambda_1 \xi_1^r, \cdots, \lambda_m \xi_m^r$ separately fixed, or at most permuting them. The latter result holds for linear substitutions in the $GF[p^n]$, if r be neither a multiple of p nor of the form p^s+1. If r be of either of these two forms, the determination of the group reduces to that of the group in the $GF[p^{2s}]$ leaving invariant $\sum_{i=1}^{m} \xi_i^{p^s+1}$. The latter group may be called the *Hyperorthogonal Group*, since those of its substitutions which belong to the $GF[p^s]$ form the orthogonal group in the $GF[p^s]$.

The hyperorthogonal group is generated by the substitutions

$$T_{i,\tau}: \quad \xi_i' = \tau\xi_i \qquad (\tau^{p^s+1}=1),$$

$$O_{ij}^{\alpha\beta}: \begin{cases} \xi_i' = \alpha\xi_i + \beta\xi_j \\ \xi_j' = -\beta^{p^s}\xi_i + \alpha^{p^s}\xi_j \end{cases} \qquad (\alpha^{p^s+1}+\beta^{p^s+1}=1),$$

except for the case $p^s = 2$, $m \geqq 3$, when a new generator is necessary, for example,

$$W: \begin{pmatrix} 1 & 1 & 1 \\ 1 & i & i^2 \\ 1 & i^2 & i \end{pmatrix} \qquad [i^2+i+1 \equiv 0 \pmod 2].$$

Those of its substitutions having determinant unity form a self-conjugate subgroup of index p^s+1. The factors of composition of this subgroup are the prime factors of d and the composite number

$$\frac{1}{d}\,[p^{sm}-(-1)^m]\,p^{s(m-1)}\,[p^{s(m-1)}-(-1)^{m-1}]$$
$$p^{s(m-2)}\cdots\,[p^{2s}-1]\,p^s,$$

where d is the greatest common divisor of m and p^s+1. The cases $m=3$, $p^s=2$; $m=2$, $p^s=2$; $m=2$, $p^s=3$ are exceptional. For $m=2$, the linear substitutions in the $GF[p^{2s}]$ which have determined unity and which leave invariant $\xi_1^{p^s+1}+\xi_2^{p^s+1}$ form a group isomorphic with the linear fractional group in the $GF[p^s]$.

§ 13. *The Hyperabelian group.*

This group consists of the totality of linear homogeneous substitutions on $2m$ indices in the $GF[p^{2n}]$ which leave invariant

$$\psi \equiv \sum_{i=1}^{m} (\xi_{2i-1}\xi_{2i}^{p^n} - \xi_{2i}\xi_{2i-1}^{p^n}).$$

The totality of substitutions with coefficients in the $GF[p^n]$ which leave ψ invariant forms a subgroup which is readily recognized as the abelian linear group on $2m$ indices in the $GF[p^n]$. Hence the name *hyperabelian.** It is easily proven to be simply isomorphic with the hyperorthogonal group on $2m$ indices in the $GF[p^{2n}]$.

* Dickson: "Concerning the four known simple groups of order 25920, with an introduction to the hyperabelian group," *Proc. Lond. Math. Soc.*, vol. 30 (1899).

§ 14. *Isomorphisms between Various Linear Groups.*

The conception of "the compounds of a given linear homogeneous group," introduced in recent papers by the writer,* has proven to be a powerful means of setting up isomorphisms. The group of quaternary linear homogeneous substitutions of determinant unity in the $GF[p^n]$ is isomorphic with a subgroup of the senary linear group leaving invariant $\xi_1\eta_1 + \xi_2\eta_2 + \xi_3\eta_3$, and hence, according as $p^n = 2^m$, $4l+1$, or $4l-1$, to the senary first hypoabelian group, the senary orthogonal group, or the group of § 10.

The simple group of order $\frac{1}{2}(p^{4n}-1)(p^{2n}-1)p^{4n}$ derived from the quaternary abelian group and that from the quinary orthogonal group, each in the $GF[p^n]$, $p>2$, are simply isomorphic.

The following four simple groups of order 25920 are simply isomorphic: †

1°. The abelian group on four indices modulo 3.

2°. The second hypoabelian group on six indices modulo 2.

3°. The orthogonal group on five indices modulo 3.

4°. The hyperabelian group on four indices in the $GF[2^2]$.

In the paper cited an abstract group, a substitution-group on 36 letters, and one on 27 letters are given, each isomorphic to the above groups of order 25920.

SHORTER NOTICES.

Leçons sur la détermination des orbites, professées à la Faculté des Sciences de Paris. Par F. TISSERAND; redigées et developpées pour les calculs numériques, par J. PERCHOT; avec une Préface de H. POINCARÉ. Paris, Gauthier-Villars et Fils. 1899. 4to, xiv + 124 pp.

TISSERAND is chiefly known outside of France for his Traité de mécanique céleste—a monument of labor undertaken and carried out amid the pressure of many duties. He had in fact an enormous capacity for work and in his numerous memoirs left few parts of theoretical astronomy untouched. The volume of 120 quarto pages before us, edited by M. Perchot, shows him working for the practical

* "Concerning a linear homogeneous group in $C_{m,q}$ variables isomorphic to the general linear homogeneous group in m variables," BULLETIN, Dec., 1898. "The structure of certain linear groups with quadratic invariants," *Proc. Lond. Math. Soc.*, vol. 30, pp. 70–98. "A new definition of the general abelian linear group," *Transactions* (read Feb. 1899).

† *Proc. Lond. Math. Soc.*, vol. 31. Abstract in BULLETIN, May, 1899, p. 384.

astronomer. The problem of the determination of an orbit from three observations is not exactly an enticing one. It is a tiresome piece of analysis which appears to offer little scope at the present time for the aspiring mathematician. In the case of a parabolic orbit the solution can even be made without successive approximations as M. Poincaré points out in the preface. In the case of an elliptic orbit, almost the only point of mathematical interest is the presence of a transcendental equation.

The treatises on this subject are well known, if not numerous. Gauss, in his Theoria motus, laid the foundations of the method at present in practical use, while Laplace a short time before had given a different form of solution. In the preface to the 'Leçons,' M. Poincaré has devoted some space to showing that Gauss's method is really equivalent to that of Laplace in spite of their apparent dissimilarity. The former has, however, been fully worked up with tables and formulæ for practical use in the great treatises of Oppolzer (of which there is a French translation) and Watson. In spite of the fact that a computer who has at his finger ends the notation and formulæ given in either of the last named volumes, will probably not care to change to another, M. Tisserand's work may nevertheless be found of use. The equations are developed quickly and easily, and moreover are all put together in a form which admits of immediate application. The young astronomer, unless he has acquired a firm grasp of his mathematics, may perhaps find it somewhat difficult to see the bearing of all the formulæ owing to the brevity of the explanations.

There are two chapters. The first on the method of Olbers for the determination of the orbit of a comet from three observations; the second on Gauss's method for the similar determination of the orbit of a planet. A numerical example of the latter, fully worked out, is set forth, showing the form in which the calculations would be actually made. Tables VIII and IX of Oppolzer's treatise are reproduced.

ERNEST W. BROWN.

Vorlesungen über mathematische Physik, Band I. Mechanik. Von GUSTAV KIRCHHOFF. 4te Auflage von W. WIEN. Teubner, Leipzig.

KIRCHHOFF's four volumes on mathematical physics which have appeared at various times since 1876 are so well known that little need be said about them here. The first volume

on general mechanics has already reached its fourth edition. The second and third editions were issued under the superintendence of the author. Dr. Wien has had the advantage of the author's manuscript notes, found after his death, in preparing the volume for the fourth edition. No changes have, however, been made beyond the correction of printer's errors and the removal of slight obscurities. One of the best testimonies that we can give to the care and ability which Kirchhoff devoted to the original volume is to say that those who possess any one of the first three issues will not find it necessary to buy this last edition.

ERNEST W. BROWN.

Leçons élémentaires sur la théorie des formes et ses applications géometriques, a l'usage des candidats a l'agregation des sciences mathematiques. Par H. ANDOYER. Paris, Gauthier-Villars et Fils, 1898. 4to, 184 pp. Lithographed.

THIS title represents accurately the contents of the book. A restricted list of topics is adequately treated. Geometrical applications are plentiful but do not encroach upon nor obscure the purely algebraic theory. The treatment is perhaps elementary, but in style simplicity is less noticeable than brevity. The work as a whole is more nearly a syllabus than a textbook for the beginner, and as a syllabus it cannot fail to become widely known and valued.

Binary and ternary forms are introduced, but of binary forms only linear, bilinear, quadric, cubic, and quartic forms are discussed; and of ternary, only linear, bilinear, and quadric. The bilinear ternary form in cogredient variables I do not remember having seen in any earlier text, although Salmon's Higher plane curves gives a brief geometrical discussion of skew reciprocity; its inclusion here together with the form bilinear in contragredient variables must be commended by geometricians. Duality as a method stands certainly on a par with projectivity.

Two points of excellence are worthy of special mention. The first is that from the outset stem forms are assumed to contain several sets of variables; and this convention is observed in the case of each particular form, an unlimited number of sets of cogredient variables being adjoined. Of course this amplifies the complete system of each stem form, but the extension is easy, since no new types occur. The second novel merit is that the term *polar* is so defined as to include such operations as Aronhold's. If so-called variables are replaced by cogredient variables, the process is ordinarily called a polar operation, and M. Andoyer sees no

reason why a different term should be needed when coefficients are replaced by cogredient coefficients. The reason why the operation is invariant is the same in both cases. As illustrating the degree of elegance attained by M. Andoyer may be cited from § 3, Chapter I, the proof (susceptible of improvement, though not of simplification in form) that absolute invariants exist; and the entire § 2 of chapter III, which sets forth the relation of a homography to a binary bilinear form.

In a text prepared especially to emphasize geometrical applications, the purely algebraic side of any problem is naturally of only secondary importance. Here the student learns to think of invariance as based on reasons rather than on specific normal forms; it is not important for the beginner to reduce the canonizant, for example, to typical form, and it is important to see clearly that every eliminant of a set of equations must be an invariant, regardless of the notation used to express it. This principle leads to a preponderance of reasoning over reckoning, and the postponement of purely algebraic problems. While complete form systems are given for each special stem form, Gordan's theorem is simply stated without proof. Nor is there any mention of Hilbert's theorems (though on p. 101 it is stated that "Le théorem de Gordan subsiste dans le domaine ternaire,") nor of the enumeration problems solved by Sylvester and Deruyts. This marks the elementary character of the work, and leaves much to be looked for from the larger compend which, the preface tells us, the author has had in preparation already for some years.

HENRY S. WHITE.

Sur les lois de réciprocité. Par X. STOUFF, professeur à la Faculté des Sciences de Besançon. Paris, Hermann, 1898. 8vo, 31 pp.

THE laws of reciprocity have been the object of numerous mathematical researches, the chief of which are the memoirs of Jacobi, Eisenstein, and Kummer in *Crelle's Journal.* Stouff proposes to apply n-dimensional geometry to the subject following the examples of Minkowski's geometry of numbers.* At bottom the question of the laws of reciprocity appears to have a natural and close connection with the fuchsian polygons of Poincaré generalized for space of any number of dimensions. The intervention of these polygons appears implicitly in Gauss's memoir on biquad-

* Minkowski, Geometrie der Zahlen, Leipzig, Teubner, 1896.

ratic residues; the works of Stieltjes appertain here also. Certain differences between the various orders of residues explain themselves by difference in properties of spaces of different dimensions. The geometric reasoning of the author presupposes familiarity with transcendental geometry on the part of the reader; where the theorems admit of immediate extension to space of any number of dimensions the exposition is made for five dimensional spaces. It is beyond the scope of this short notice to recapitulate the results, which, in the nature of the subject, resist compression.

E. O. LOVETT.

Leçons sur la théorie analytique des équations aux dérivées partielles du premier ordre. Par ÉTIENNE DELASSUS, chargé de conférences à l'Université de Lille. Paris, Hermann, 1897. 8vo, 88 pp.

THE theory of partial differential equations has reached a high degree of perfection, but the theory in its present form lacks unity, its various parts resulting from particular properties and remaining almost independent of one another.

Thus the integration of linear systems rests on a partial extension of the theorem of Cauchy; in non-linear systems it is necessary to distinguish two cases according as the unknown function does or does not enter, since the method of Jacobi and Mayer experiences modifications in passing from one to the other; further the method of Lie is demonstrated only for systems in which the unknown does not figure, and is the consequence of a theorem which results from the theory of characteristics or from a method of Jacobi.

The object of the memoir of Delassus is to unify this system of details by determining a canonical form altogether general and establishing a general existence theorem. In the case of systems in which the unknown does not enter, this canonical system coincides with the form in involution, but it is no longer the same when the unknown enters explicitly. Upon this general canonical form Delassus establishes a fundamental existence theorem which he calls for short the generalized theorem of Cauchy; the demonstration of it yields the following theorem which plays a capital rôle throughout the theory: The integration of a system of partial differential equations of the first order having but one unknown can always be referred to that of a single equation of the first order. This theorem was already known for the case where the unknown function is absent, having been demonstrated by Lie by means of the general theory of characteristics and then by Mayer who employed a method of integration of Jacobi.

The author employs the generalized theorem of Cauchy to formulate the generalized problem of Cauchy which, if solved, would effect the integration completely; to study singular solutions which he classifies according as they are simply, doubly, and so on to n-ply singular; and finally to investigate precisely the transformation which changes a system where the unknown is present into one where it is absent.

The theorem on the reduction to a single equation furnishes immediately a method of integration of linear systems, the method of Mayer, the simplest known. For the integration of non-linear systems the author presents the theory of the complete integral in the ordinary manner, but adds the solution of the problem of Cauchy by means of such an integral without the geometric considerations relative to characteristics.

The exposition of the method of Jacobi and Mayer is made without using any algebraic properties of the bracket expressions and without any reference to the presence or absence of the unknown; moreover Delassus shows that the method leads always to a complete integral. He finds finally the method of Lie as an immediate consequence of the above reduction theorem, and without raising any question relative to the unknown function.

A list of forty examples of simultaneous systems forms the second part of this most valuable contribution to the theory of the linear partial differential equation.

E. O. LOVETT.

NOTES.

THE Chicago Section of the AMERICAN MATHEMATICAL SOCIETY will meet at the University of Chicago on Thursday and Friday, December 28 and 29 next. Titles and abstracts of papers to be read should be in the hands of the Secretary of the Section not later than December 12.

AMONG the officers of the American association for the advancement of science for the coming year are: Professor R. S. WOODWARD, Columbia University, president; Professor CHARLES BASKERVILLE, University of North Carolina, general secretary; Professor ASAPH HALL, JR., University of Michigan, vice-president of section A, mathematics and astronomy; Dr. W. M. STRONG, Yale University, secretary of section A.

THE meeting of the British association for the advancement of science was held at Dover, September 13–21. Professor J. H. POYNTING was president of the mathematical section, and Mr. E. T. WHITTAKER secretary. The following papers were presented to the section: Report of the committee appointed to calculate tables of certain integrals; Report of the committee appointed to calculate tables of certain mathematical functions; Mr. FRANCIS GALTON, F.R.S., "The median estimate;" Professor A. R. FORSYTH, F.R.S., "A system of invariants for parallel configurations in space;" Professor J. D. EVERETT, F.R.S., "On the notation of the calculus of differences;" Professor A. C. DIXON, "On the partial differential equation of the second order;" Professor A. R. FORSYTH, F.R.S., "On singular solutions of ordinary differential equations;" Professor IRVING STRINGHAM, "On the fundamental differential equations of geometry;" Mr. E. T. WHITTAKER, "Report on recent progress in the problem of three bodies;" Professor E. O. LOVETT, "An application and interpretation of infinitesimal transformations;" Lieut. Col. A. J. C. CUNNINGHAM, "On Fermat's numbers."

THE German mathematical association held its annual meeting at Munich from the 17th to the 23d of September in connection with the association of scientists and physicians. The preliminary programme includes the following papers in the order of announcement:—K. HENSEL, Berlin, "On the analytico-arithmetical theory of algebraic functions of two variables;" K. HEUN, Berlin, "The kinetical problems of scientific technology;" E. STUDY, Greifswald, "Geometry of dynames;" R. MEHMKE, Stuttgart, and J. BAUSCHINGER, Berlin, "Discussion of the decimal division of angles and time;" H. WEBER. Strassburg, and G. HAUCK, Berlin, "On the organization of university mathematical instruction to accord with the new Prussian examination requirements;" K. DÖHLEMANN, Munich, "On hyperboloidal straight lines;" F. ENGEL, Leipzig, "On two remarkable groups in space of five dimensions;" P. GORDAN, Erlangen, "On symmetric functions;" D. HILBERT, Göttingen, "On the number concept;" "On Dirichlet's principle;" J. HORN, Charlottenburg, "Divergent series in the theory of differential equations;" G. KOHN, Vienna, "On the geometry of Klein's group of 360 collineations and 360 correlations;" M. LERCH, Freiburg, Switzerland, "Arithmetical considerations on infinite series;" L. MAURER, Tübingen, "Invariance under special groups of linear substitutions;" F.

MEYER, Königsberg, "On the notion of equivalence of geometrical propositions;" M. NOETHER, Erlangen, "Riemann's lectures of 1861–62 on abelian functions;" E. SCHIMPF, Bochum, "Introduction of a measure of convergence and divergence in the theory of infinite processes (series, products, continued fractions, etc.);" L. SCHLESINGER, Klausenburg, "On Riemann's posthumous paper on the theory of linear differential equations;" J. SOMMER, Göttingen, "On quadratic manifoldnesses in five-dimensional space;" A. SOMMERFELD, Clausthal, "On Weierstrass's criteria in the calculus of variations;" E. v. WEBER, Munich, "A fundamental classification of differential problems." Pedagogical papers from RUDEL, of Nürnberg, SCHOTTEN, of Halle, and TREUTLEIN, of Karlsruhe, were also announced.

ON the seventeenth of last June the statue erected to the memory of CARL FRIEDRICH GAUSS and WILHELM WEBER was unveiled at Göttingen with appropriate ceremonies. The monument in bronze, on a granite base, represents Gauss seated in an arm-chair and Weber standing by his side and apparently discussing with him their joint invention, the electric telegraph. It is the work of the Berlin sculptor Hartzer. The principal address of the celebration was delivered by Professor Woldemar Voigt who sketched in brief and pregnant words the scientific career of the two men. A large number of delegates from various scientific institutions and other guests were present at the unveiling and the succeeding banquet. An exhibition of manuscripts by Gauss and Weber, of instruments used by them, and of other relics was arranged in the Aula of the University and attracted much attention. The Festschrift published in commemoration of the day contains two valuable memoirs, one by Professor Hilbert, "On the foundations of euclidean geometry," the other by Professor Wiechert, "On the theory of electrodynamics." The University of Göttingen took this opportunity to confer the honorary degree of doctor of philosophy on Professor G. H. Darwin, of Cambridge, Professor Geitel, of Wolfenbüttel, Professor J. Hadamard, of Paris, Professor Lorentz, of Leiden, Professor E. H. Moore, of Chicago, Professor Righi, of Bologna, and Professor von Sterneck, of Vienna.

THE international congress of philosophy in connection with the Paris exposition of 1900 will be held from the 2d to the 7th of August. The section devoted to the history of the sciences has made liberal provision in its programme for the discussion of the fundamental notions of mathematics.

THE Paris academy of sciences has been authorized to increase the number of its national and foreign correspondents from one hundred to one hundred and sixteen.

A MATHEMATICAL society has been organized at Liverpool, England, to be known as the Liverpool mathematical society. Mr. Walter Stott is honorary secretary.

THE London mathematical society has decided to issue its *Proceedings* in two volumes per annum.

M. GAUTHIER-VILLARS, of Paris, announces the early appearance of the following mathematical works:—A two volume treatise on the theory of forms and higher analytic geometry, by ANDOYER; an elementary treatise on the theory of numbers, by CAHEN; a treatise on geometrical optics, by WALLON; the scientific works of G. ROBIN, including the memoirs on the theory of functions founded exclusively on the idea of number, and those on mathematical physics and general thermodynamics, edited by L. RAFFY; an index to the *Comptes rendus* from 1881 to 1895, including volumes 92 to 121.

THE fall announcements of M. Hermann, of Paris, include Recherches sur les fonctions de variables réelles, by R. BAIRE; Sur les fondements de la théorie des ensembles transfinis, by G. CANTOR, translation by F. MAROTTE, of the two well known memoirs of the author published in the *Mathematische Annalen*; Leçons nouvelles sur les applications géométriques du calcul différentiel, by W. de TANNENBERG.

MESSRS. Carré and Naud, of Paris, have a history of mathematics by BOYER in the press, and an annuaire of mathematicians in preparation.

PROFESSOR R. S. BALL is engaged on a treatise on the theory of screws which is to be considerably larger than his former book on that subject. It will be published by the Cambridge university press.

THE second volume of Professor A. R. FORSYTH'S theory of differential equations is in the press.

The following courses of lectures have been announced for the winter semester 1899–1900:—

UNIVERSITY OF BERLIN.—By Professor H. A. SCHWARZ: Analytical geometry, four hours; Maxima and minima, two hours; Elliptic functions, four hours; Colloquia, two hours; Mathematical seminar, two hours.—By Professor E. R. HOPPE: Analytical geometry, four hours; Differential cal-

culus and theory of series, four hours.—By Professor J. KNOBLAUCH: Differential calculus, four hours; Applications of elliptic functions, four hours; Exercises in differential calculus, one hour.—By Professor LEHMANN-FILHÉS: Integral calculus, four hours; Method of least squares, one hour.—By Professor G. FROBENIUS: Theory of determinants, four hours; Theory of numbers, four hours; Mathematical seminar, two hours.—By Professor K. HENSEL: Synthetic geometry, four hours; Geometry of numbers, three hours; Exercises in the geometry of numbers, one hour.—By Professor G. HETTNER: Theory of probability and of errors, four hours.

UNIVERSITY OF BRESLAU.—By Professor J. ROSANES: Exercises in mathematical seminar, one hour; Differential calculus and elements of integral calculus, four hours; Elements of the theory of determinants, two hours.—By Professor R. STURM: History of mathematics, one hour; Mathematical seminar, two hours; Theory of geometrical correspondences second part, three hours; Non-euclidean geometry, two hours.—By Professor F. LONDON: Introduction to the theory of functions, three hours; The technology of insurance, two hours.

UNIVERSITY OF ERLANGEN.—By Professor P. GORDAN: Differential and integral calculus, four hours; Algebra, four hours; Seminar, three hours.—By Professor M. NOETHER: Analytical geometry, four hours; Differential geometry, four hours; Mathematical exercises.

UNIVERSITY OF GÖTTINGEN.—By Professor F. KLEIN: Mechanics of deformable bodies, four hours; Mathematical seminar, two hours.—By Professor D. HILBERT: Integral calculus, four hours; Curvature of surfaces, two hours; Number-concept and quadrature of circle, two hours; Exercises in mathematical seminar, one hour.—By Dr. F. SCHILLING: Descriptive and projective geometry, four hours; Exercises in descriptive geometry, four hours; Exercises in differential calculus, one hour.—By Dr. G. BOHLMANN: Chapters in the mathematics of insurance, two hours; The mathematical foundations of insurance, three hours; Mathematical exercises in seminar, two hours.—By Dr. ZERMELO: Theory of functions, four hours.—By Dr. SOMMER: Theory of numbers, four hours; Exercises in the theory of numbers, one hour.

UNIVERSITY OF GREIFSWALD.—By Professor W. THOMÉ: Analytical geometry, two hours; Theory and applications

of determinants, two hours; Exercises in mathematical seminar, two hours.—By Professor E. STUDY: Mechanics, four hours; Applications of the infinitesimal calculus to geometry, four hours; Exercises in seminar, one hour.

UNIVERSITY OF KIEL.—By Professor L. POCHHAMMER: Plane analytical geometry, three hours; Theory of definite integrals, three hours; Mathematical seminar, one hour.—By Professor P. STÄCKEL: Integral calculus, three hours; Higher geometry, three hours; Seminar, one hour.

UNIVERSITY OF KOENIGSBERG.—By Professor F. MEYER: Integral calculus, three hours; Exercises to preceding, one hour; Elliptic functions, four hours; Seminar, one hour.—By Professor A. SCHOENFLIES: Determinants, one hour; Projective geometry, four hours; Exercises to preceding, one hour; General introduction to higher mathematics, two hours.—By Professor L. SAALSCHÜTZ: Differential equations, three hours; Exercises in differential equations, one hour.—By Dr. T. VAHLEN; Calculus of variations, three hours; Exercises, one hour.

UNIVERSITY OF HALLE-WITTENBERG.—By Professor G. CANTOR: Analytical mechanics, four hours; Chapters of the infinitesimal calculus, two hours; Exercises in seminar, two hours.—By Professor A. WANGERIN: The most important partial differential equations of mathematical physics, one hour; Theory of surfaces and space curves, four hours; Synthetic geometry, three hours; Mathematical seminar, two hours.—By Professor V. EBERHARD: Chapters of analytical geometry, one hour; Theory of numbers, four hours.

UNIVERSITY OF HEIDELBERG.—By Professor L. KOENIGSBERGER: Analytical mechanics, four hours; Calculus of variations, differential equations, two hours; Elliptic functions, two hours; Seminar, two hours.—By Professor M. CANTOR: Infinitesimal calculus, four hours; Exercises in the calculus, one hour; Political arithmetic, two hours.—By Professor F. EISENLOHR: Theoretical optics, four hours; Infinitesimal calculus, five hours; Potential, two hours.—By Professor K. KOEHLER: Analytical geometry of space, three hours.—By Professor G. LANDSBERG: Descriptive geometry, three hours; Introduction to the theory of algebraic equations, two hours.

UNIVERSITY OF JENA.—By Professor J. THOMAE: Algebraic analysis, four hours; Mathematical geography, three

hours; Seminary, two hours.—By Professor G. FREGE: Analytical mechanics, four hours; Symbolic algebra, two hours.

UNIVERSITY OF LEIPZIG.—By Professor A. MAYER: Higher analytical mechanics, four hours; Exercises to the preceding, one hour.—By Professor C. NEUMANN: Infinitesimal calculus, four hours; Seminar, one hour.—By Professor O. HÖLDER: Introduction to the theory of functions of a complex variable, four hours; Chapters in the theory of substitution groups and equations, two hours; Seminar, one hour.—By Professor F. ENGEL: Analytical mechanics, four hours; Theory of continuous transformation groups, two hours; Seminar, one hour.—By Dr. F. HAUSDORFF: Chapters of higher geometry, four hours.—By Dr. G. KOWALEWSKI: Algebraic invariant theory.

UNIVERSITY OF MUNICH.—By Professor G. BAUER: Plane analytical geometry; Conduction of heat in fixed bodies; Seminar.—By Professor F. LINDEMANN: Differential calculus; Abelian functions; Mathematical seminar on theory of automorphic functions and application of elliptic functions.—By Professor A. PRINGSHEIM: Introduction to the theory of analytic functions; Elementary theory of infinite algorithms (series, products, continued fractions).—By Dr. H. BRUNN: Introduction to the mathematical foundations of natural science.—By Dr. K. DÖHLEMANN: Descriptive geometry and exercises; Synthetic geometry and exercises.—By Dr. E. v. WEBER: Introduction to analysis; Introduction to the Cauchy-Riemann theory of functions.

THE London mathematical society has conferred its De Morgan medal on Professor W. BURNSIDE for his mathematical researches in the theory of groups of a finite order.

THE Adams prize offered by the University of Cambridge for an essay on the theory of the aberration of light has been divided between Mr. J. LARMOR, fellow of St. John's college, senior wrangler 1880, and Mr. G. T. WALKER, fellow of Trinity college, senior wrangler in 1889.

THE Madrid academy of sciences recently announced the results of the competition for its mathematical prize set in 1897. There were three competitors. Professors GINO LORIA, of the University of Genoa, and GOMES TEIXEIRA, of the Polytechnic of Oporto, each received a prize, and JOAQUIN DE VARGAS Y AGUIRRE, of Salamanca, an honorable mention.

THE honorary degree of doctor of laws has been recently conferred on Professor A. R. FORSYTH by Dublin University, and that of doctor of civil law on Professor SIMON NEWCOMB, by Oxford University.

PROFESSOR L. FUCHS has been made rector of the University of Berlin for the current year, and Professor E. NETTO has received the same appointment at the University of Giessen.

MR. H. M. MACDONALD, of Clare College, Cambridge, fourth wrangler in 1889, has been appointed to the University lectureship in mathematics made vacant by Professor A. E. H. LOVE's recent resignation to accept the Sedleian professorship in mathematics at Oxford University.

LORD KELVIN has resigned the chair of natural philosophy in Glasgow University after a tenure of fifty-three years.

DR. MILTON UPDEGRAFF, professor of astronomy in the University of Missouri, has been appointed to a professorship in mathematics in the United States naval observatory.

DR. L. E. DICKSON has resigned an assistant professorship of mathematics in the University of California to accept an associate professorship in the University of Texas.

MR. PETER FIELD, fellow in Cornell University, has been appointed professor of mathematics in Carthage College.

THE following have recently qualified as privatdocenten: Dr. GRASSMANN, at the University of Halle; Dr. KOWALEWSKI, at the University of Leipzig; Dr. J. GOETTLER, at the University of Munich.

MESSRS. T. BIRTWHISTLE, of Pembroke College, and R. P. PARANJPYE, of St. John's College, were bracketed for the senior wranglership of this year at Cambridge University. The Smith's prizes for 1899 were awarded to Messrs. W. H. AUSTIN and G. W. WALKER, of Trinity College, senior and fourth wranglers in 1897. Mr. PARANJPYE is the first native of India to attain the honor and Mr. BIRTWHISTLE is the first member of Pembroke College to achieve it since Sir GEORGE STOKES who was senior wrangler in 1841.

AT Emmanuel College, Cambridge, Mr. G. T. BENNETT, senior wrangler, 1890, has been elected to a senior fellow-

ship, and Mr. H. S. CARSLAW, fourth wrangler 1894, to a junior fellowship.

MR. A. H. WILSON, instructor in mathematics at Princeton University, has been granted a year's leave of absence, and Dr. L. W. REID has been appointed to an instructorship in the same institution.

THE following mathematicians have been elected to fellowships in the Reale Accademia dei Lincei: Professor P. TARDY and G. VERONESE, as ordinary fellows; Professor G. RICCI, as corresponding fellow; Professor G. MITTAG-LEFFLER and J. WEINGARTEN, as foreign fellows.

PROFESSORS F. KLEIN and W. NERNST, of Göttingen, have been elected to foreign membership in the academy of sciences of Budapest.

AT its recent decennial celebration Clark University conferred the honorary degree of doctor of laws on five scientists among whom were Professors PICARD, of Paris, and BOLTZMANN, of Vienna.

MR. J. D. THOMPSON, of Trinity College, Cambridge, has been elected to an assistant professorship of mathematics in the University of West Virginia.

DR. F. H. SAFFORD, of Harvard University, has been made assistant professor of mathematics at the University of Cincinnati.

MR. HOWARD OPDIKE has been promoted to an assistant professorship of mathematics at Union College.

DR. H. Y. BENEDICT and MR. I. N. PUTNAM have been appointed instructors in mathematics at the University of Texas.

THE Right Rev. CHARLES GRAVES, Lord Bishop of Limerick, professor of mathematics at Trinity College since 1843, died recently at the age of eighty-seven years.

THE deaths are announced of Professor ROBERT WILHELM BUNSEN, August 16th, 1899, at the age of eighty-eight years; of M. GRIESS, one of the editors of *L'éducation mathématique*, at the age of forty-two years.

NEW PUBLICATIONS.

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